ZBIGNIEW DWORECKI, ANDRZEJ FISZER, MARIUSZ ŁOBODA, JACEK PRZYBYŁ
Institute of Agricultural Engineering
Agricultural University of Poznan, Poland

CALCULATION OF TRACTOR AND AGRICULTURAL MACHINES MOMENTS OF INERTIA

S u m m a r y

Determination of tractor moments of inertia is not so easy. There are known few methods of indirect measurement of moment of inertia, but the hanging tractor is necessary in this methods. Suspension of the tractor is a long shot. This is the reason, that most of authors assume the quantity of moment of inertia only intuitionally, without measurements, in their computation. Tractor moments of inertia can be evaluated through "geometric-mass" tractor model. Main tractor elements, like corps, wheels, cabin, wheel reduction gears, half-shafts, can be admitted by non complicated geometric body, like cylinder, ring, cone, rectangular, torus, plate, box constructed from plate. Dimensions and weightiness of this geometric body must be the same that tractor units. The main moments of inertia of this solid are easily calculated. Steiner principle provide calculation main moments of inertia of all tractor body. These moments of inertia were applied in mathematical models of the tractor, built for eigenfrequencies calculation. Tractor vibrations were measured for verification of the models. These measured frequencies were similar to frequencies calculated from received models. It means, that suggested method of tractor moments of inertia determination, is accurate. This easy method allows to leave out big mistakes in calculation of tractor free vibrations frequencies.

OBLICZANIE MOMENTÓW BEZWŁADNOŚCI CIĄGNIKÓW I MASZYN ROLNICZYCH

S t r e s z c z e n i e

Znanych jest kilka metod wyznaczania momentów bezwładności maszyn. Wymagają one jednak zawieszania ciągnika w różnych pozycjach, co jest pracochłonne i wymaga stosowania odpowiednich urządzeń. To powoduje, że wielu autorów przyjmuje wartości momentów intuicyjnie, bez pomiarów, i stosuje je do swoich obliczeń. W pracy przedstawiono metodę obliczania momentów bezwładności, polegającą na podziale ciągnika na elementy, które można zastąpić prostymi bryłami. Wymiary i masy tych brył muszą być takie same jak wymiary i masy zastępowanych fragmentów ciągnika. Momenty bezwładności tych brył są łatwe do obliczenia. Momenty bezwładności całego ciągnika obliczono dzięki zastosowaniu zasady Steinera. Obliczone momenty bezwładności oraz inne potrzebne parametry zastosowano w matematycznym modelu ciągnika, zbudowanym w celu obliczenia częstotliwości jego drgań własnych. Przeprowadzono pomiary drgań własnych ciągnika w celu porównania ich z częstotliwościami obliczonymi. Częstotliwości te były bardzo zbliżone do siebie, co może świadczyć o poprawności obliczenia momentów bezwładności. Ta prosta metoda wyznaczania momentów bezwładności pozwala uniknąć dużych błędów podczas obliczania częstotliwości drgań własnych.

1. Introduction

The farm tractor is the farmer's basic tool. Consequently, it is also his frequent workplace. In performing their work in the field, tractor operators are exposed to dust, noise, warm - cold weather (temperature), precipitation (rain) and mechanical vibrations. The tractor cab may eliminate dust, noise and precipitation. Air-conditioned cab may eliminate effect of temperature, and, provided the cab is suspended on springs, partially eliminate vibrations. The vibrations affecting the driver may be generated by the engine, machinery attached to the tractor and tractor wheels (collisions with obstacles). Vibrations generated by the engine and machinery attached to the tractor are insignificant in present engines and machines. Vibrations generated by tractor wheels may be detrimental to human body organs, because the frequency of tractor vibration can be the same that eigenfrequency of human body organs. Particularly hazardous are the high-amplitude and low-frequency vibrations which can cause damage to many human body organs. Frequencies of tractor vibrations are a combination of tractor eigenfrequencies and depend on excitation. Tractor eigenfrequencies we can to get to know if we build mathematical model of the tractor.

2. Model of the tractor

A dynamic model may be created by using many methods. There is no need to prove that the finite element method is too accurate for our purposes. It is so because of the big discrepancy between the rigidities of tires and the driver's seat spring-suspension system and the rigidities of other parts of the tractor. Therefore such units are analysed with the use of the concentrated mass method. Literature (Mitschke, 1989; Göhlich, 1991; Graef, 1976) abounds with models of vehicles, including models of tractors. However sometimes, such models do not account for the differences between a typical truck and tractor such as the lack of suspension of the rear axle and the degree of swinging of the tractor front axle. The simplest models of the tractor which may be accepted, can be created in lateral plane and front plane of the tractor (fig. 1 and 2). The mode (shape) of vibrations in lateral plane tractor model is vertical and pitch. The mode of vibrations in front plane is vertical and roll.

The mathematical formula of lateral plane tractor model shown on fig. 1 is:

$$\omega_{1,2b}=\sqrt{\frac{1}{2}\left(\frac{k_1 l^2+k_2(a-l)^2}{J_b}+\frac{k_1+k_2}{m}\right)\mp\sqrt{\frac{1}{4}\left(\frac{k_1 l^2+k_2(a-l)^2}{J_b}-\frac{k_1+k_2}{m}\right)^2+\frac{(k_2(a-l)-k_1 l)^2}{J_b\cdot m}}}$$

The mathematical formula of front plane tractor model shown on fig. 2 is:

$$\omega_{1c}=\sqrt{\frac{2k_{1,2}}{m}}\,, \qquad \omega_{2c}=r\sqrt{\frac{k_{1,2}}{2J_c}}\,.$$

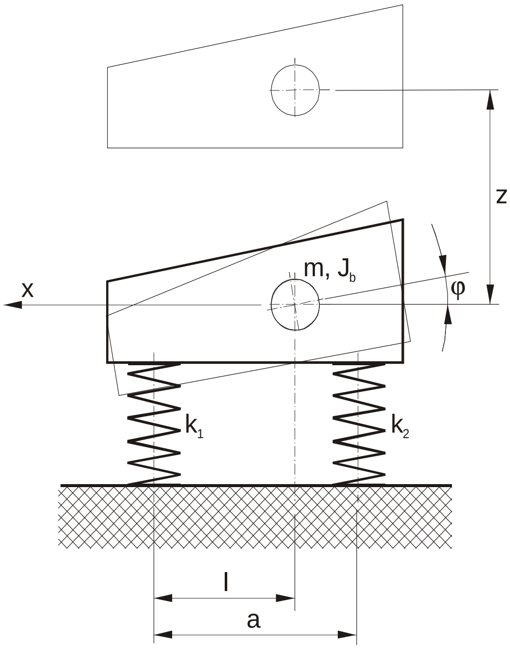

Fig. 1. Lateral plane tractor model: m - mass of the system, J_b - pitch moment of inertia of the system, k_1 - the sum of front tyres stiffness, k_2 - the sum of rear tyres stiffness, „a” - wheel base (rear axle to front axle), l - front axle to centre of gravity

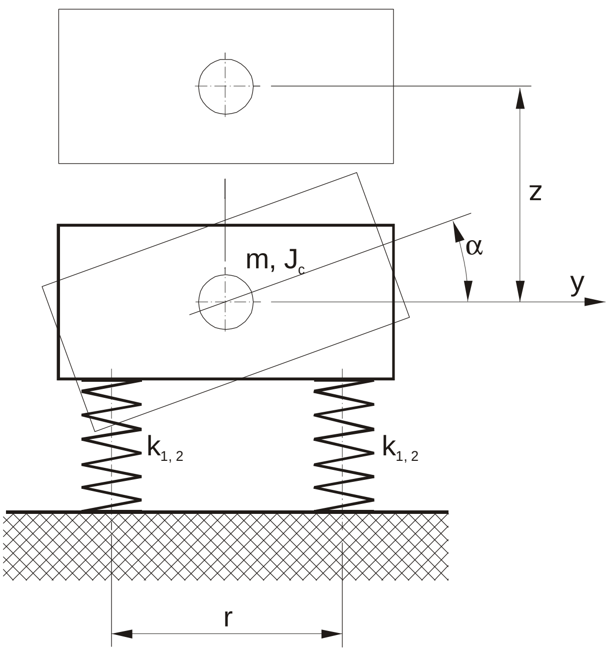

Fig. 2. Front plane tractor model: m - mass of the system, J_c - roll moment of inertia of the system, $k_{1,2}$ - the sum of one front and one rear tyre stiffness, r - track of wheels

For calculation the frequency from presented formulas, it is necessary to know the quantity of all the parameters from right side of the equations.

The mass and the geometric parameters „a” (wheel base, rear axle to front axle) and „r” (track of wheels) may be found by using factory specifications or by weighing and measuring. The „l” distance (front axle to centre of gravity) may be calculated by using factory specifications of axle loads. The measuring of vertical stiffness of front and rear tyres is also not difficult. The rigidity of tires may be calculated by changes of the wheels load with simultaneous measurements of ground reaction with the accuracy of 1N and the resulting deflections with 0,01 mm accuracy.

3. Calculation of moments of inertia

The mass moment of inertia of the tractor relative to the front-back axis and left-right axis, cutting through the mass centre, can be determined in a variety of ways. The methods based on the principle of the pendulum (physical or torsional or three point suspension) require that the tractor be suspended few times in mid-air, and are difficult to carry out in practice. The moment of inertia can be found more easily by measuring the frequencies of the tractor's lateral and front vibrations. Yet, the moment of inertia determined in such a way cannot be used for calculating previously measured frequencies. In such a case, the model would not be verifiable. This is the reason, that most of authors assume the quantity of moment of inertia only intuitionally, without measurements, in their computation (Sakai, 1999; Kumar et al., 2001) They accept the numbers 3000, 4000 kg*m^2, with accuracy 1000 kg*m^2. This is incorrect.

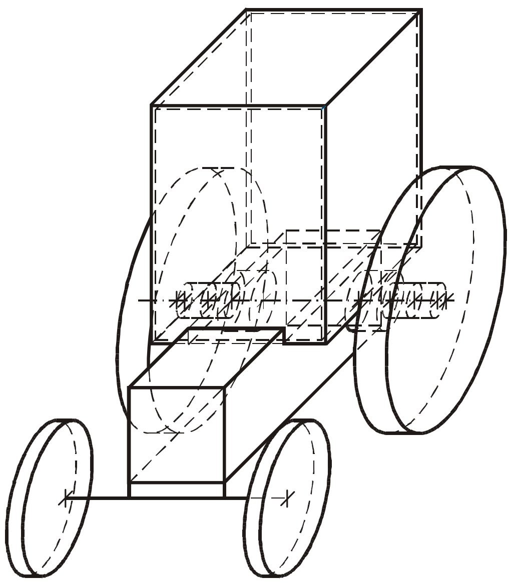

Fig. 3. Tractor “geometric-mass” model

Tractor and agricultural machine moments of inertia can be evaluated through "geometric-mass" tractor or machine model. The main tractor units, like a tractor frame, wheels, cabin, wheel reduction gears, half-shafts, can be represented by non complicated geometric bodies, like cylinder, ring, cone, rectangular, torus, plate or box constructed from plate. Also machine units can be represented by these bodies. Dimensions and weightiness of these geometric bodies must be the same as tractor units. The main moments of inertia of this block are easily calculated. The Steiner principle provides calculation main moments of inertia of all tractor body.

For example, we will find parameters for URSUS 1002, a popular model of tractor in Poland. The geometric body composition of this tractor is shown on fig. 3.

The mass and dimensions of simple solids were quantified from known (tab. 1) and measured (tab. 2) parameters of U 1002 tractor.

Adequately to tractor "geometric-mass" model, the tractor is structured from:

- ► tractor frame - rectangular 0.6x0.6x2.785 [m], 2983 kg,
- ► driver cab - box constructed from plate 0.02 m, length 1.7 m, width 1.3m, height 1.5 m, 325 kg,
- ► front wheels - cylinder: width 0.1 m, diameter 0.88 m, 48 kg,
- ► rear wheels - cylinder: width 0.2 m, diameter 1.50 m, 227 kg,
- ► wheel reduction gears - cylinder: width 0.2 m, diameter 0.34 m, 50 kg,
- ► half-shafts - cylinder: width 0.3 m, diameter 0.20 m, 26 kg.

It was not difficult to calculate the moments of inertia of these parts.

The Steiner principle provides calculation main moments of inertia of all tractor body:

J_b=3447 kg*m^2 J_c=1074 kg*m^2

4. Discussion

The main question is about exactitude of moments of inertia valuation.

Error made during valuation the parameter is difference among the accepted and real value. It is unjustly to say generally about exactitude of this method of moments of inertia valuation. Every time, the exactitude of valuation depends on the partition of the machine on geometric body exactitude. In our case, we assume 0.05-0.18 [m] inaccuracy of dimensions depend of geometric body. Valuated error of pitch moment of inertia was 4.8% and valuated error of roll moment of inertia was 9.8%.

Table 1. Selected parameters of U 1002 tractor, known from factory specifications

PARAMETER	**QUANTITY**
Rear axle to front axle	2.385
Front wheels track [m]	1.350; **1.500**; 1.650; 1.800
Rear wheels track [m]	**1.500**; 1.650; 1.800; 1.875
Length without three-point linkage system [m]	3.945
Width (without ballast) when wheels track is 1,500 [m]	1.930
Tractor mass without ballast - front axle load [kg] - rear axle load [kg]	4010 1398 2612
Driver cab mass [kg]	325

For presented axle loads, l distance is 1.554 m

Table 2. Selected measured parameters of U 1002 tractor

PARAMETER	**QUANTITY**
Rear axle level [m]	0.75
Breadth of rear wheel bangle [m]	0.38
Least diameter of rear wheel bangle [m]	0.88
Front axle level [m]	0.44
Breadth of front wheel bangle [m]	0.18
Rear axle to cab floor (vertically) [m]	0.2
Rear axle to cab geometric centre (longitudinally) [m]	0.45
Cab height [m]	1.5
Cab length -on the roof [m] -on the floor [m]	1.5 2.2
Cab breadth -on the roof [m] -on the floor [m]	1.25 1.35
Rear wheel mass [kg]	227
Front wheel mass [kg]	48

These moments of inertia and another measured parameters were applied in the mathematical models of the tractor, shown on fig. 1 and 2, build for eigenfrequencies calculation. The frequencies calculated from the model were:

- ► for pitch vibration: 3.82 Hz,
- ► for roll vibration: 4.47 Hz,
- ► for vertical vibration: 3.16 Hz (lateral plane), 3.22 Hz (front plane).

The tractor vibrations were measured for verification of the models (Dworecki, 2003). The eigenfrequencies were determined by experimental modal analysis. In this system-analysis technique the dual-channel FFT analyser (Discrete Fast Fourier Transformation, Brüel & Kjær) was used to measure the ratio of the response to the measured input force. The frequency response function (FRF) measurement removes the force spectrum from the data and describes the inherent structural response between the measurement points.

From the collected FRF measurements which were made at 8 defined points on the tractor, was build up the set of its response for vertical, lateral and longitudinal excitation. Observation of vibrations mode pointed on eigenfrequencies:

- ►for pitch vibration: 3.86 Hz,
- ►for roll vibration: 4.61 Hz,
- ►for vertical vibration: 2.39 Hz.

It was surprising, that the pitch and roll measured frequencies were so similar to frequencies calculated from received models. The bigger difference between calculated and measured vertical frequencies is because the observed frequency shape for 2.39 Hz, was only similar to vertical vibration, however most similar from all modal frequencies. It means, that suggested method of tractor moments of inertia determination, is accurate, but every time, the exactitude of valuation depends on the partition of the machine on geometric body exactitude. This easy method allows avoiding a big mistake in evaluation of the frequencies of tractor vibrations.

5. Bibliography

[1] Dworecki Z. (2003): Investigations of Tractor with Agricultural Implement Vibrations, Oriented on Effect on Tractor Driver (in Polish). Inżynieria Rolnicza, No 5 (47): 1-96.

[2] Gőohlich H., Kaplick Ch., Pickel P, Siefkes T. (1991): Fahrdynamik von Traktoren. Landtechnik 7/8: 335-340.

[3] Gőohlich H., Kaplick Ch., Pickel P, Siefkes T. (1991): Fahrdynamik von Traktoren. Teil 2: Bewertung von Fahrzeugschwingungen, Schwingungsersatzysteme und Maβnahmen zur Schwingungsisolation. Landtechnik 9: 434-437.

[4] Graef M. (1976): Technische Mőglichkeiten zum Senken der Schwingungsbelastung auf fahrenden Arbeitsmaschinen. Grundl. Landtechnik Bd.26. Nr. 2: 56-63.

[5] Kumar A., Mahajan P., Mohan D., Varghese M. (2001): Tractor Vibration Severity and Driver Health: a Study from Rural India. Journal of Agricultural Engineering Research. Vol. 80, No. 4: 313-328.

[6] Mitschke M. (1989): Dynamic of automobile (in Polish). WKŁ, Warszawa.

[7] Sakai K. (1999): Theoretical Analysis of Nonlinear Dynamics and Chaos in Bouncing Tractor. Journal of Japanese Society of Agricultural Machinery. Vol. 61, No. 6: 65-71.