**Semion IVANOV [1], Volodimir BULGAKOV [2], Tadeusz PAWŁOWSKI [3]**
[1] Latvia University of Agriculture, Research Institute of Agricultural Machinery, semjons@apollo.lv)
[2] Ukrainian Academy of Agrarian Sciences,
[3] Przemysłowy Instytut Maszyn Rolniczych w Poznaniu, tadek@pimr.poznan.pl

# THEORETICAL INVESTIGATION OF THE PROCESS OF DIGGING ROOTS VIBRATION METHOD

*Summary*

*A new mathematical model is developed which describes the process of direct sugar beet extraction from soil, carried out by interaction of the vertical impact force and the tractive effort, which are transmitted to the root crop from the vibratory digging tool. Sets of differential equations have been obtained the solution of which enables to determine the law of the root crop movement during their direct vibrational extraction.*

## 1. Introduction

The use of sugar beet vibrational extraction from soil has a number of essential advantages in comparison with other ways. It is characterized by lesser damage of the roots, lower losses of the crop during the harvest period, more intensive cleaning of the root crops from the clods of soil, lesser blocking up of the working channel of the digger with soil and plant residues. Therefore this technological process requires detailed analytical research, further development, and manufacturing application of advanced vibratory digging tools.

## 2. Materials and method

At first let's carry out the necessary formalization of the technological process to be considered. Despite the fact that the process of sugar beet extraction from soil takes a short interval of time (the forward speed of the root diggers can reach 2 m/s), all the process can be conditionally divided into separate interconnected successive operations [1]. As it was noted above, extraction is possible only in the case of a symmetric grip of the root crop by the digging tool with simultaneous transformation of the vibrations of the root crop into its angular vibrations around a conditional point of fixing of the root crop in soil.

At the first stage of extraction, and especially at the first vibrations, the restoration force at the angular vibrations and therefore, its moment in relation to the conditional point of fixing will be maximal. That's why the angle of inclination of the root crop will be sufficiently small and full (or partial) restoration of its vertical position owing to the forward movement of the digger will be possible. Nevertheless, due to the action of the root crop, the forward vibrations together with the soil surrounding it, the compactness of the soil will decrease, and the restoration force at the angular vibrations will decrease too. So, with each successive vibration the angle of inclination of the root crop will increase, and restoration of the previous position will decrease. The root crop will be loosened around the conditional point of its fixing with gradual increase of its inclination angle forward in relation to the course of the digger. This will lead to the loss of the contact between the root crop and the loose soil in the direction of the movement of the digger, beginning from the top part of the root crop conic surface, gradually approaching its conditional point of fixing in soil. So, as it was stated above, it follows that the destruction of the root crop contact with soil occurs simultaneously in two directions – along the forward movement of the digger and in the perpendicular direction (along the full depth of the root crop position in soil). Thus the forces of root crop contacts with soil and the elasticity forces of soil will gradually decrease to such a minimal value at which the vibrational processes are transformed into the processes of root crop continuous movement upwards and forward – along the movement of the digger, and also into continuous root crop rotation around their centers of mass. The elasticity forces of soil will pass into the resistance forces of the loose soil during the root crop movement through the working channel of the digger. After that the stage of the sugar beet direct extraction from the soil starts [2].

In order to develop a mathematical model, first of all we shall make an equivalent scheme of the root crop interaction with the working surfaces of the vibratory digging tool during the root crop direct extraction (fig.). Let's present the vibratory digging tool in the form of two coupled digging surfaces (wedges) $A_1B_1C_1$ and $A_2B_2C_2$, each of which having an inclination in space at angles $\alpha$, $\beta$, $\gamma$ and which are so located in relation to each other that a working channel is formed with its rear part narrowed. The pointed wedges produce vibrational movements in the longitudinal-vertical plane (the mechanism of the vibratory movement of the shares is not shown) with corresponding amplitudes and frequencies. The direction of the forward movement of the vibratory digging tool is shown by an arrow. The projections of points $B_1$ and $B_2$ on axis $O_1\ y_1$ are marked accordingly by points $D_1$ and $D_2$.

Let's consider how the root crop interacts with the surfaces of wedges $A_1B_1C_1$ and $A_2B_2C_2$ in the corresponding points. The root crop is approximated by a cone-shaped body, and gripping of the root crop by the digging tool occurs symmetrically from its both sides. Let's assume further that the working surface of wedge $A_1B_1C_1$ forms a direct contact with the root crop at point $K_1$, and surface $A_2B_2C_2$ at point $K_2$. The lines pass through points $K_1$ and $K_2$ of the root crop contact, and points $B_1$ and $B_2$ form a section with the sides of wedges $A_1C_1$ and $A_2C_2$ corresponding to points $M_1$ and $M_2$. Thus, $\delta$ is a dihedral angle ($<B_1M_1D_1$) between the lower base of $A_1D_1C_1$ and the working surface of the wedge $A_1B_1C_1$, or, accordingly, a dihedral angle ($<B_2M_2D_2$) between the lower base of $A_2D_2C_2$ and the working surface of the wedge $A_2B_2C_2$. Let's show the forces which arise owing to the interaction of the root crop with the vibratory digging tool. Interaction of the vibratory

digging tool with the vertical impact force $\overline{Q}_{tr}$ proceeds according to a harmonious law of the form:

$$Q_{tr} = H \sin \omega t \qquad (1)$$

where $H$ – the amplitude of the impact force; $\omega$ – the frequency of the impact force.

This force plays a basic role during the loosening of soil in the zone of the digger working channel, and the root crop extraction. A specific impact force $\overline{Q}_{tr}$ is applied to the root crop from its two sides, and on the scheme it is presented by two components $\overline{Q}_{tr.1}$ and $\overline{Q}_{tr.2}$. These forces are applied accordingly to points $K_1$ and $K_2$ at the distance $h$ from the conditional point of fixing $O$, causing vibrations of the root crop in longitudinal-vertical planes which destroy the contact of the root crop with soil and create the required conditions for the extraction of root crops from soil. As the grips of the root crops are symmetric, it is obvious that there will be the following correlation:

$$Q_{tr.1} = Q_{tr.2} = \frac{1}{2} H \sin \omega t \qquad (2)$$

Let's decompose the given forces into normals $\overline{N}_1$ and $\overline{N}_2$ and tangential components $\overline{T}_1$ and $\overline{T}_2$, as it is shown in the figure. As the vibratory digger moves forward in the direction of axis $O_1x_1$ in relation to ~~a~~ the root crop which is fixed in soil, and during the moment of the grip of the root crop by the digging tool - in the direction of axis $O_1x_1$, the motive forces $\overline{P}_1$ and $\overline{P}_2$ operate too. Let's decompose forces $\overline{P}_1$ and $\overline{P}_2$ into two components: normals $\overline{L}_1$ and $\overline{L}_2$, and tangents $\overline{S}_1$ and $\overline{S}_2$ to surfaces $A_1B_1C_1$ and $A_2B_2C_2$, accordingly. Besides, at the points of contact $K_1$ and $K_2$ the forces of friction $\overline{F}_{K1}$ and $\overline{F}_{K2}$ act accordingly, which counteract the root crop sliding along the working surface of wedges $A_1B_1C_1$ and $A_2B_2C_2$ during its grip by the vibratory digging tool. The vectors of these forces are directed opposite to the vector of the relative speed of a root crop sliding along the surface of the wedges. The root crop sliding along the surface of the wedges can move in the direction of forces $\overline{T}_1$, $\overline{T}_2$ (parallel lines $B_1M_1$ and $B_2M_2$) and in the direction, opposite to the action of forces $\overline{S}_1$, $\overline{S}_2$, due to the motion resistance force of soil.

The vector of the relative speed of the root crop sliding along the surface of the wedges can be decomposed into components in the directions specified above. So, the force of friction $\overline{F}_{K1}$ can also be decomposed into two components: $\overline{F}_1$ in a direction, opposite to vector $\overline{T}_1$, and $\overline{E}_1$ – in the direction of vector $\overline{S}_1$. Similarly, the force of friction $\overline{F}_{K2}$ can be decomposed into two components: $\overline{F}_2$ – in a direction, opposite to vector $\overline{T}_2$, and $\overline{E}_2$ – in the direction of vector $\overline{S}_2$.

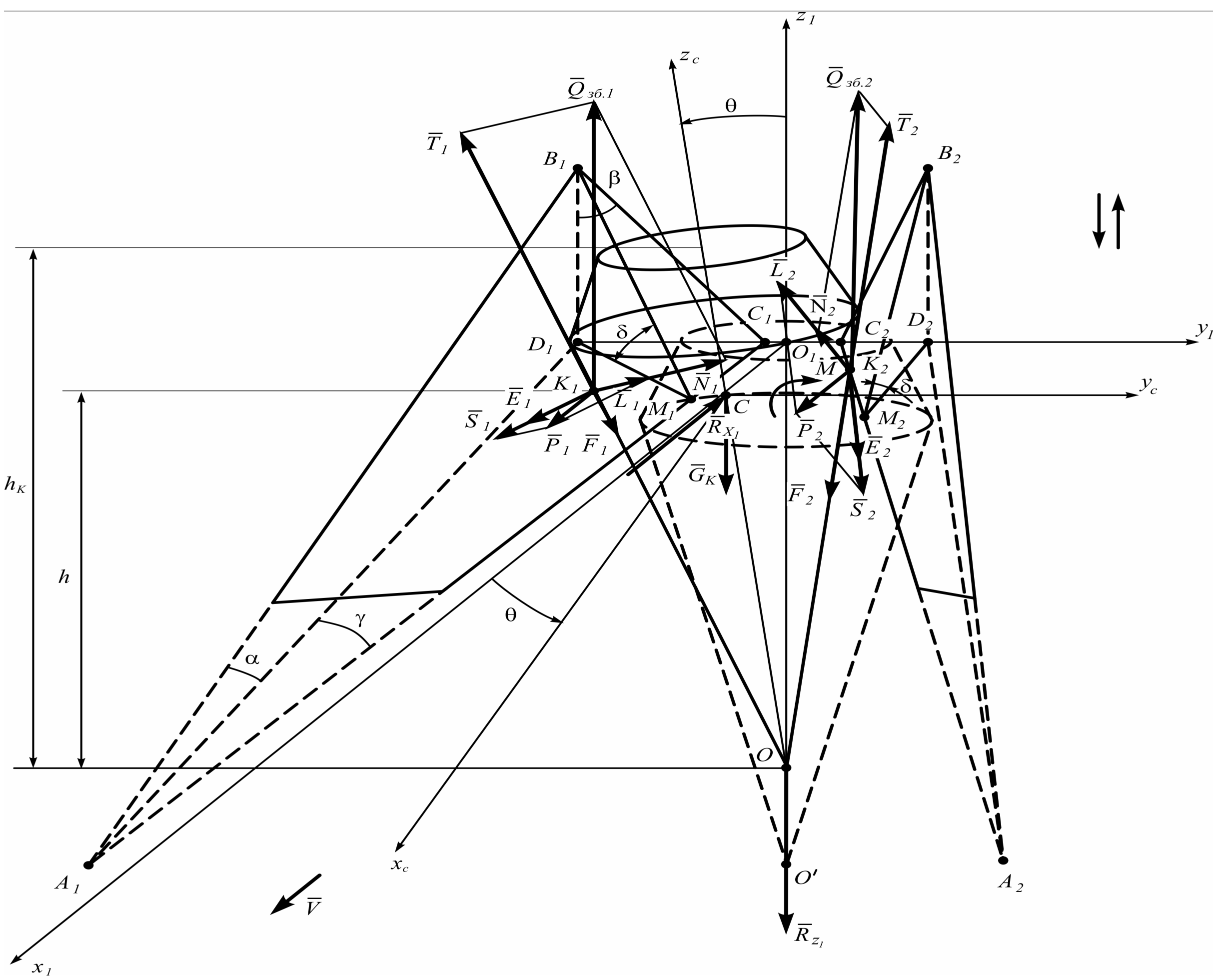


*Figure. The equivalent scheme of root crop vibrational extraction from soil*

It is obvious that $F_1=F_2$, $E_1=E_2$. In the center of the root crop mass (point $C$) the force of the root crop mass operates $\overline{G}_k$.

The forces of resistance of the loose soil during the root crop movement through the working channel of the digger in the direction of axes $O_1x_1$ and $O_1z_1$ are designated as $\overline{R}_{x1}$ and $\overline{R}_{z1}$ accordingly.

### 3. Research results

During the direct root crop extraction from soil the rotation of the root crop around its center of mass (point $C$) will be carried out under the action of a pair of resistance forces of the loosened soil. We shall designate the moment of this pair of forces as $M$.

At the direct root crop extraction it is possible to consider the forces of resistance of the loosened soil depending on the speed of the root crop movement in the loosened soil or as a first approximation – as simple constants. Therefore, to simplify the mathematical model, we shall consider forces $\overline{R}_{x1}$, $\overline{R}_{z1}$ and the moment of the pair $M$, as constants.

At first let's make differential equations for the movement of the center of a root crop mass (point $C$), i.e. the forward movement of a root crop along axes $O_1x_1$ and $O_1z_1$. Considering the scheme of forces given above, the differential equation of the movement of the centre of the root crop mass in the vector form during their direct extraction will have the form:

$$m_k\overline{a} = \overline{N}_1 + \overline{N}_2 + \overline{L}_1 + \overline{L}_2 + \overline{F}_1 + \\ + \overline{F}_2 + \overline{E}_1 + \overline{E}_2 + \overline{G}_k + \overline{R}_{z1} + \overline{R}_{x1} \quad (3)$$

where $\overline{a}$ – acceleration of the movement of the root crop mass center.

Since the process of extraction, as it has been specified above, occurs by symmetric gripping of the root crop by means of the digging tool, the root crop movement along the working channel of the digger occurs actually in longitudinal-vertical planes (planes $x_1O_1z_1$). This is why the vector equation (3) is reduced to a set of two equations in the projections to axes $Ox_1$ and $Oz_1$.

After the definition of the values of all the forces which enter the vector equation (3), and their projections to axes $Ox_1$ and $Oz_1$ we shall produce the following two sets of differential equations:

$$\left.\begin{aligned} \ddot{x}_1 = \frac{1}{m_k}\Bigg[\frac{\cos\delta\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}} + f\cos^2\delta\,\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\gamma + \\ + f\cos\delta\cos\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\cos\gamma\Bigg]H\sin\omega t + \frac{2}{m_k}\times \\ \times\Bigg[\frac{\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}} + f\sin^2\gamma\,\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\cos\delta + \\ + f\sin\gamma\cos\gamma\cos\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\Bigg]P_1 - \frac{R_{x1}}{m_k}, \\ \ddot{z}_1 = \frac{1}{m_k}\Bigg[\frac{\cos\delta\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}} - f\cos\delta\,\sin\left(\gamma+\frac{\alpha_{K_1\max}}{2}\right)\sin\delta\Bigg]H\sin\omega t + \\ \frac{2}{m_k}\Bigg[\frac{\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}} - f\sin\gamma\,\sin\left(\gamma+\frac{\alpha_{K_1\max}}{2}\right)\sin\delta\Bigg]P_1 - \frac{R_{z_1}}{m_k} - g, \\ \omega t\in[2k\pi,\, 2(k+1)\pi],\; k=0,1,2,... \end{aligned}\right\} \quad (4)$$

$$\left.\begin{aligned} m_k\ddot{x}_1 = \frac{2P_1\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}} + 2fP_1\sin^3\gamma\cos\delta + fP_1\sin 2\gamma\cos\gamma - R_{x1}, \\ m_k\ddot{z}_1 = \frac{2P_1\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}} - 2fP_1\sin^2\gamma\sin\delta - G_k - R_{z1}, \end{aligned}\right\} \quad (5)$$

$$\omega t\in[(2k-1)\pi,\; 2k\pi,\;],\; k=1,2,...$$

Thus the set of differential equations (4) describes the process of direct vibrational extraction of the root crops from soil (i.e. the distance at which the periodic impact force acts upon the root crop), and the set of differential equations (5) describes the process of the root crop extraction from soil when the impact force does not act upon it, i.e. the same vibratory digging tool can realize in different time intervals the process of the root crop digging as the usual share digger. Let's solve the obtained sets of differential equations.

For the given sets of differential equations (4), (5) the initial conditions will be the following: at $t = 0$:

$$\dot{x}_1 = 0, \qquad \dot{z}_1 = 0 \quad (6)$$

$$x_1 = x_{10}, \qquad z_1 = -\frac{1}{3}h_k \quad (7)$$

The set of differential equations (4) is a set of linear differential equations of the second order. As it is known, it is solved in quadratures. For the simplification of recording of the set of differential equations (4), let's write:

$$\frac{1}{m_k}\left[\frac{\cos\delta\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}}+f\cos^2\delta\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\gamma+\right. \tag{8}$$

$$\left.+f\cos\delta\cos\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\cos\gamma\right]=\phi_1,$$

$$\frac{2}{m_k}\left[\frac{\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}}+f\sin^2\gamma\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\cos\delta+\right. \tag{9}$$

$$\left.+f\sin\gamma\cos\gamma\cos\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\right]=\psi_1,$$

$$\frac{1}{m_k}\left[\frac{\cos\delta\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-f\cos\delta\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\delta\right]=\phi_2, \tag{10}$$

$$\frac{2}{m_k}\left[\frac{\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-f\sin\gamma\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\delta\right]=\psi_2. \tag{11}$$

Considering expressions (8) – (11), the set of differential equations (4) will obtain the form:

$$\left.\begin{aligned}\ddot{x}_1&=\phi_1 H\sin\omega t+\psi_1 P_1-\frac{R_{x1}}{m_k},\\ \ddot{z}_1&=\phi_2 H\sin\omega t+\psi_2 P_1-\frac{R_{z1}}{m_k}-g.\end{aligned}\right\} \tag{12}$$

Let's integrate the set of differential equations (12). After twofold integration and finding any arbitrary constants we obtain the following solutions of differential equations (4) in a final form:

$$\left.\begin{aligned}\dot{x}_1&=-\frac{\phi_1 H}{\omega}\cos\omega t+\psi_1 P_1 t-\frac{R_{x1}t}{m_k}+\frac{\phi_1 H}{\omega},\\ \dot{z}_1&=-\frac{\phi_2 H}{\omega}\cos\omega t+\psi_2 P_1 t-\frac{R_{z1}t}{m_k}-gt+\frac{\phi_2 H}{\omega}.\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}x_1&=-\frac{\phi_1 H}{\omega^2}\sin\omega t+\frac{\psi_1 P_1 t^2}{2}-\frac{R_{x1}t^2}{2m_k}+\frac{\phi_1 Ht}{\omega}+x_{10},\\ z_1&=-\frac{\phi_2 H}{\omega^2}\sin\omega t+\frac{\psi_2 P_1 t^2}{2}-\frac{R_{z1}t^2}{2m_k}-\frac{gt^2}{2}+\frac{\phi_2 Ht}{\omega}-\frac{1}{3}h_k.\end{aligned}\right\} \tag{14}$$

The sets of equations (13) and (14) describe the laws of the change in speed and the shift of the centre of the root crop mass during its direct extraction from soil. From the second equation of the set (14) it is possible to define the time $t$ of the root crop direct extraction from soil. For this purpose it is necessary to substitute in the left part of the specified equation value $z_1 = 0$ and to solve the obtained equation in relation to $t$. As the equation is transcendental, it is impossible to obtain an analytical expression for definition $t$; nevertheless it can be solved on the computer by means of the known numerical methods. The calculated mean $t_1$ can be applied to the definition of the unit productivity for the root crop extraction by means of vibratory digging tools.

Let's solve the set of differential equations (5). To simplify the recording of the given set, let's write:

$$\frac{1}{m_k}\left(\frac{2\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}}+2f\sin^3\gamma\cos\delta+f\sin 2\gamma\cos\gamma\right)=\psi_1', \tag{15}$$

$$\frac{1}{m_k}\left(\frac{2\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-2f\sin^2\gamma\sin\delta\right)=\psi_2' \tag{16}$$

In view of expressions (15), (16) the set of differential equations (5) will assume the form:

$$\left.\begin{aligned}\ddot{x}_1&=\psi_1' P_1-\frac{R_{x_1}}{m_k},\\ \ddot{z}_1&=\psi_2' P_1-\frac{G_k}{m_k}-\frac{R_{z_1}}{m_k},\end{aligned}\right\} \tag{17}$$

$$\omega t\in\left[(2k-1)\pi,\ 2k\pi,\ \right],\ k=1,2,...$$

After twofold integration of the set of equations (17) and finding any arbitrary constants we shall obtain the final set of differential equations (5) in the final form:

$$\left.\begin{aligned} \dot{x}_1 &= \psi_1' P_1 t - \frac{R_{x_1}}{m_k} t, \\ \dot{z}_1 &= \psi_2' P_1 t - \frac{G_k}{m_k} t - \frac{R_{z_1}}{m_k} t, \end{aligned}\right\} \quad (18)$$

$$\omega t \in [(2k-1)\pi,\ 2k\pi,\ ],\ k = 1, 2,...$$

$$\left.\begin{aligned} x_1 &= \psi_1' P_1 \frac{t^2}{2} - \frac{R_{x_1} t^2}{2m_k} + x_{10}, \\ z_1 &= \psi_2' P_1 \frac{t^2}{2} - \frac{G_k t^2}{2m_k} - \frac{R_{z_1} t^2}{2m_k} - \frac{1}{3} h_k, \end{aligned}\right\} \quad (19)$$

$$\omega t \in [(2k-1)\pi,\ 2k\pi,\ ],\ k = 1, 2,...$$

Sets of equations (18) and (19), accordingly, describe the laws of the change in speed and the movement of the centre of the root crop mass during their direct extraction from soil in the absence of the impact force.

Let's set up a differential equation of the root crop rotation around their center of mass, or around ~~a~~ the conditional axis $Cy_c$ which passes through the centre of the mass (point $C$) on the parallel axis $O_1y_1$. According to the equation will have a general form:

$$I_{y_c} \frac{d^2\theta}{dt^2} = M_{y_c}^e, \quad (20)$$

where $\theta$ – the angle of the root crop rotation around axis $Cy_c$; $I_{yc}$ – the moment of inertia of the root crop in relation to axis $Cy_c$; $M_{y_c}^e$ – the moment of rotation around axis $Cy_c$ (the sum of the moments of all external forces which act upon the root crop, in relation to axis $Cy_c$).

The moment of inertia $I_{yc}$ of the root crop in relation to axis $Cy_c$ is defined according to by such an expression:

$$I_{y_c} = \left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon\right) m_k h_k^2 \cdot \quad (21)$$

After substituting expressions (2), (21) in the differential equation (20) and carrying out the necessary transformations we shall obtain the differential equation of the rotation of the root crop around axis $Cy_c$ during direct vibrational extraction from a soil (i.e. at the action of the impact force on it) which has the form:

$$\begin{aligned} &\left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon\right) m_k h_k^2 \frac{d^2\theta}{dt^2} = -H\left(-h_k + h - z_1\right)\sin\theta \sin\omega t + 2P_1 \cos\theta\left(-h_k + h - z_1\right) + \\ &+ 2\left(\frac{1}{2} f H \cos\delta \sin\omega t + f P_1 \sin\gamma\right)\sin\left(\gamma + \alpha_{K1\ \max}\sin\omega t\right)\cos\varepsilon\left(-h_k + h - z_1\right)\sin\theta + \\ &+ 2\left(\frac{1}{2} f H \cos\delta \sin\omega t + f P_1 \sin\gamma\right)\cos\left(\gamma + \alpha_{K1\ \max}\sin\omega t\right)\cos\gamma\left(-h_k + h - z_1\right)\cos\theta - M, \\ &\omega t \in [2k\pi,\ (2k+1)\pi],\quad k = 0, 1, 2,... \end{aligned} \quad (22)$$

The differential equation of the root crop rotation around axis $Cy_c$ at usual extraction (i.e. in the absence of impact force), has the form:

$$\begin{aligned} &\left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon\right) m_k h_k^2 \frac{d^2\theta}{dt^2} = 2P_1 \cos\theta\left(-h_k + h - z_1\right) + 2 f P_1 \sin^2\gamma \times \\ &\times \cos\varepsilon\left(-h_k + h - z_1\right)\sin\theta + f P_1 \sin 2\gamma \cos\gamma\left(-h_k + h - z_1\right)\cos\theta - M, \end{aligned} \quad (23)$$

$$\omega t \in [(2k-1)\pi,\quad 2k\pi],\qquad k = 1,\ 2,...$$

Let's analyze the obtained differential equations (22) and (23). The differential equation (22) is nonlinear. It is possible to solve it by approximated numerical methods using a computer, and in each step of the application of a numerical algorithm it is necessary to find value $z_1$ from the second equation of the set (14) for the corresponding moment of time $t_k$. The differential equation (23), which includes a variable quantity $z_1$, is also nonlinear, and for each moment of time $t_k$ it is necessary to define value $z_1$ from the second equation of the set (19).

Thus, it is finally possible to regard that a mathematical model is developed of the process of direct sugar beet extraction from soil by vibrational digging. The obtained results enable to define kinematic modes of the root crop vibrational digging considering the constructive parameters of the vibratory digging tools.

## 4. Conclusions

1. Two sets of differential equations which describe plane-parallel motion of a root crop in soil during its direct extraction carried out by interaction of the vertical impact force, which is transmitted the root crop from the vibratory digging tool, and the tractive effort, which arises owing to longitudinal movement of the digger.
2. These differential equations provide an opportunity to find out the law of the root crop movement in a longitudinal-vertical plane during their direct extraction from soil.
3. The obtained results enable also to define the kinematic modes of the root crop vibrational digging without causing damage to the roots and to find rational constructive parameters of the vibrational digging tool.

## 5. References


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