Anna KUCZUK, Janusz POSPOLITA
Opole University of Technology, Poland
Department of Thermal Engineering and Industrial Equipment
e-mail: a.kuczuk@po.opole.pl ; j.pospolita@po.opole.pl

MODELLING OF WATER FLOW IN SOIL

Summary

This paper presents a mathematical model of water flow in soil. It is based on Richards equation taking into consideration a negative source member which determines water uptake by crops. In the performed calculations, different and varied weather conditions were assumed, which were then applied to differently adopted boundary conditions. In case of precipitation, Dirichlet condition of full saturation of soil with water was assumed for the surface, and Neuman condition was assumed at the lower boundary of calculation area. In drought period, Neuman type condition, in a form of a function, determines water loss through soil surface, and Dirichlet condition determines the pressure of capillary rise. In the model, hydraulic conductivity of soil and suction pressure were made depending on its local moisture level. Soil profiles from an exemplary farm were analysed. Equations of the mathematical model were solved with the finite difference method. Non-stationary problem was solved with an explicit method. Numerical calculations were made with an software program written in FORTRAN compilation language. Results of moisture level variations in soil are presented for various simulated weather conditions. The target model and program developed will be expanded for the needs of the analysis of migration of nitrogen compounds in soil.
***Key words**: soil, water, modelling*

MODELOWANIE RUCHU WODY W GLEBIE

Streszczenie

W pracy przedstawiono model matematyczny ruchu wody w glebie. Został on oparty na równaniu Richardsa przy uwzględnieniu ujemnego członu źródłowego, charakteryzującego pobór wody przez rośliny wybranej uprawy. W przeprowadzonych obliczeniach przyjęto różne i zmienne warunki pogodowe, które przełożone zostały na różnie przyjmowane warunki brzegowe. W przypadku opadów przyjmowano na powierzchni warunek Dirichleta pełnego nasycenia gleby wodą i warunek Neumanna na dolnym brzegu obszaru obliczeniowego. W okresie suszy warunek typu Neumanna w postaci funkcji określa ubytek wody przez powierzchnię gleby, a warunek typu Dirichleta określa ciśnienie podsiąku kapilarnego. W modelu przewodność hydrauliczną gleby i ciśnienie ssące uzależniono od miejscowej jej wilgotności. Do analizy wzięto profile glebowe z przykładowego gospodarstwa rolnego. Równania modelu matematycznego rozwiązano metodą różnic skończonych. Niestacjonarne zagadnienie rozwiązano metodą jawną. Obliczenia numeryczne przeprowadzono w oparciu o autorski program napisany w języku kompilacyjnym FORTRAN. Przedstawiono wyniki obliczeń zmian wilgotności w glebie dla różnych, symulowanych warunków pogodowych. Docelowo opracowany model i program zostanie rozbudowany dla potrzeb analizy migracji związków azotu w glebie.
***Key words**: gleba, woda, modelowanie*

1. Introduction

Presence of water in soil and its flow are critical for the life and growth of plants and for farming activity. Water penetrates into the soil from precipitation, moisture in air, by capillary rise or artificial irrigation. It leaves the soil by evaporating, is drawn by plant roots and infiltrates to lower layers of the soil, to ground waters.

Physical and chemical properties of soil, in particular strong dependency of hydraulic conductivity of unsaturated soil and suction pressure on soil moisturising level, cause that water can persist in the soil for long. In case of dry soil, however, the capillary rise can be even from the depth of 2 meters [4, 10].

Water flow in soil allows transport of ingredients which are essential for the life of plants, e.g. fertilising components. Such flow causes also spreading of pollutants in the soil and their infiltration to ground water [8].

Modelling of water flow in soil is a complex issue. This is due to both, considerable non-linearity of the equation describing it, and non-linearity of the relationships between water moisture level and its hydraulic conductivity. It is also difficult to clearly formulate a boundary condition on soil surface to cover different ways of water exchange between soil and atmosphere. Also the plants, at different vegetation stages, draw different amounts of water from soil. This amount also depends on weather conditions [2].

Note also that soil is structurally varied throughout its volume and at individual levels of soil profiles. It makes modelling the water flow in soil difficult. Such modelling requires making frequent simplifying assumptions and, in the absence of own analyses and measurements, adopting various literature data, e.g. related to the relationship between soil moisture level and its physical and chemical properties, such as suction pressure of hydraulic conductivity.

A different issue is to obtain numerical solutions to the conditions which, in real time, can last several months.

Despite the above difficulties, modelling of water flow in soil has been a subject of numerous papers and monographs [1, 8, 9, 10]. It is an important issue. It provides theoretical information about vegetation of plants. The model is also useful, when proper equations are applied, for analysing spread of contamination in soil.

This study aims to create a mathematical model and a computer program for analysing water flow in soil in various hydrological conditions and for various physical parameters of soil.

2. Modelling of water flow

Mathematical model of water flow in soil was based on Richards equation [4, 10] which, when transformed, takes the form (1):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\theta)\frac{\partial h_s(\theta)}{\partial z}\right) - \frac{\partial K(\theta)}{\partial z} - S(\theta) \tag{1}$$

In this equation, z is a ccoordinate, which refers to the depth of the soil and t is a time. The parameter θ is the soil moisture defined as a part of soil volume filled with liquid in relation to total soil volume. K(θ) is hydraulic conductivity of unsaturated soil. In saturated soil, it reaches the boundary value determined by soil type. h_s value is the suction pressure of soil. Its value, dependent on soil moisture θ, is expressed by the formula (2):

$$h_s(\theta) = -\frac{1}{\gamma}\left(\left(\frac{1}{s}\right)^{\frac{1}{m}} - 1\right)^{\frac{1}{n_g}} \tag{2}$$

where: $\gamma = 0.2$ [cm^{-1}], $n_g = 7.0$, $m = 1- 1\cdot n_g^{-1}$ are empirical constants. The value s is defined as follows (3):

$$s = \frac{\theta - \theta_{min}}{\theta_{max} - \theta_{min}} \tag{3}$$

In the above formula, θ_{max} and θ_{min} denote respectively: maximum soil moisture (soil saturated with water) and its minimum moisture. It has to be added that there are numerous other relationships described in literature which associate suction pressure of soil with its moisture.

The relationship between hydraulic conductivity and moisture of soil is presented with the equation (4) of [8], which is derived from [12]:

$$K(\theta) = K_s s^{\eta}\left(1 - \left(1 - s^{\frac{1}{m}}\right)^m\right)^2 \tag{4}$$

In the above equation, K_s is the hydraulic conductivity of saturated soil, and $\eta = 0.5$ and m, are empirical constants. The values of these constants are given the results of the calculations.

The source member S(θ) in the equation (1) determines water consumption by the plants. As in the paper [4], that value is expressed by the equation (5):

$$S(\theta) = \alpha(\theta)\frac{E_0}{l} \tag{5}$$

In the equation (5), α(θ) is a value that takes constant value for a specified soil moisture range. According to [4], after converting pF = log $|h_s|$ to values θ, it yields:

for 0,44 > θ > 0,375 α(θ) = 0,
for 0,375 > θ > 0,15 α(θ) = 1,
for 0,15 > θ > 0,08 α(θ) = (θ – 0,08) / 0,07,
for 0,08 > θ α(θ) = 0.

The value E_o determines transpiration of water from plants, a1 the depth of their roots. In this paper, the water uptake assumed was from 0.05 cm^3 to 0.1 cm^3 per cm^2 of soil surface in 24 hours.

The initial and boundary conditions depend on the issue analysed. For example, as an initial condition, uniform, low moisture of soil was adopted (θ = 0.08). As the upper boundary condition (Dirichlet type) at the surface, constant soil moisture was adopted (θ = 0.3), resulting from constant water supply from the atmosphere. Water infiltrates deep into the soil which results from the assumed low moisture of soil and its suction force.

Neumann condition $\frac{\partial \theta}{\partial t} = 0$ was assumed as the second, lower boundary condition. It indicates stabilisation of water flow, which is effected only by the gravity force.

3. Calculation methods

Equations of the mathematical model were solved with the finite difference method, with the aid of an original program written in FORTRAN algorithmic language. The finite difference method is successfully applied to solve numerous tasks in fluid mechanics [3, 5, 6].

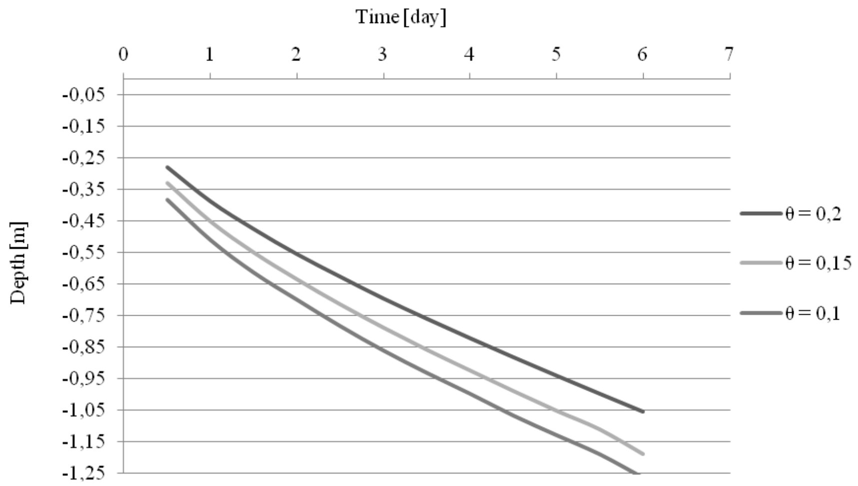
To obtain solution of the equation (1) of the mathematical model in time, the first-order explicit method was used. Nonlinear diffusion member in this equation was approximated using central differences. Finally, the differential equation (6) below corresponds to the differential equation (1):

In the first equation, Δt and Δz are respectively the time and spatial steps on a differential lattice and the indices n and z denote respectively: time level n –"new", n-1 – "old" and node on the axis z. The explicit method assumed imposed limitations as to the maximum value of the step in time Δt. This is related to stability of the calculation procedure [6, 7]. In the calculations performed, Δt of the order of 0.005 s was assumed. It makes the calculations last longer, especially because issues are modelled which last for weeks in real time.

4. Results of calculations and discussion

The first issue studied was the saturation of soil with water. As an initial condition, uniform, low moisture of soil was adopted (θ = 0.08). From the moment t = 0, a boundary condition on the surface was adopted, assuming the constant, close to the maximum soil moisture θ = 0.3, which might have been an effect of many days of rain. Neumann condition $\frac{\partial \theta}{\partial z} = 0$ was adopted at the depth 1.25 m, which corresponds to full water outflow through a drainage system. Figure 1 shows soil moisture variations in time for three selected values θ: 0.2; 0.15 and 0.10. In case of the constants assumed, $K_s = 1.157\cdot10^{-6}$, $\gamma = 0.2$, and $n_g = 7.0$, saturation of soil with water practically takes place after 6 days.

$$\theta_z^{n+1} = \theta_z^n + \Delta t\left(\left(\left(K_{s,z+1}^n + K_{s,z}^n\right)\cdot\frac{h_{s,z+1}^n - h_{s,z}^n}{2\Delta z} - \left(K_{s,z}^n + K_{s,z-1}^n\right)\cdot\frac{h_{s,z}^n - h_{s,z-1}^n}{2\Delta z}\right) - \frac{K_{s,z+1}^n - K_{s,z-1}^n}{2\Delta z} + S_z^n\right) \tag{6}$$



Source: Own study / Źródło: badania własne

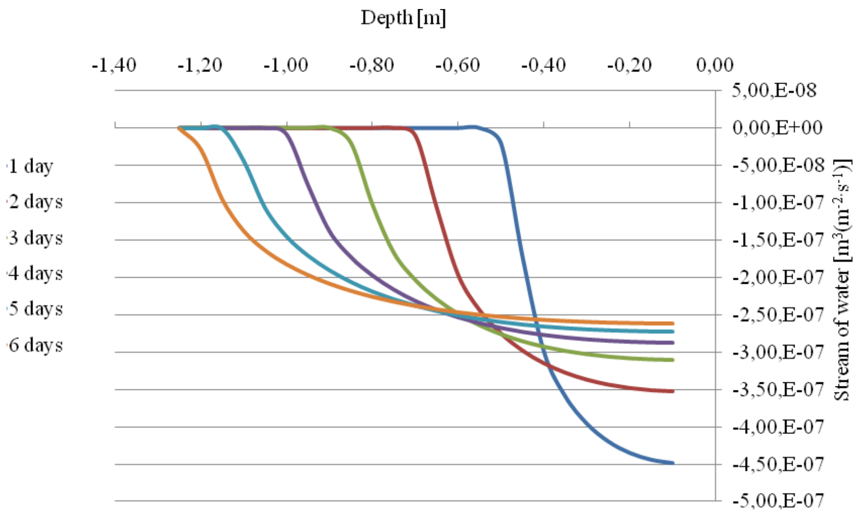
Fig. 1. Changes in soil moisture in time (K_s = 1,157·10^{-6}, γ = 0.2 and n_g = 7,0)
Rys. 1. Zmiany wilgotności gleby w czasie (K_s = 1,157·10^{-6}, γ = 0,02, oraz n_g = 7,0)

Figure 2 shows the values of water stream fw at various depths in the soil vs. the time. The values of water stream and soil moisture indicate that the frontline of moisture increase in the soil moves gradually deep into the soil. The speed of the frontline movement depends on the soil type, which is related to the values of hydraulic conductivity of unsaturated soil K(θ). It changes by several orders of magnitude along with the change of soil moisture. It is a fundamental mechanism that holds water in the soil, when its surface moisture radically drops.

Figure 3 shows soil moisture variations assuming the capillary rise. It was initially assumed that the entire layer of soil was dry (θ = 0.08). At the depth 1.25 m, constant moisture of soil occurs θ = 0.30. In the surface layer from 0 m to 0.3 m, there are roots of plants drawing the water. Intensity of water uptake by plant roots was assumed according to the equation (4). Figure 3 shows the variation of soil moisture vs. time, and Figure 4 presents water stream variation in the soil layer analysed.

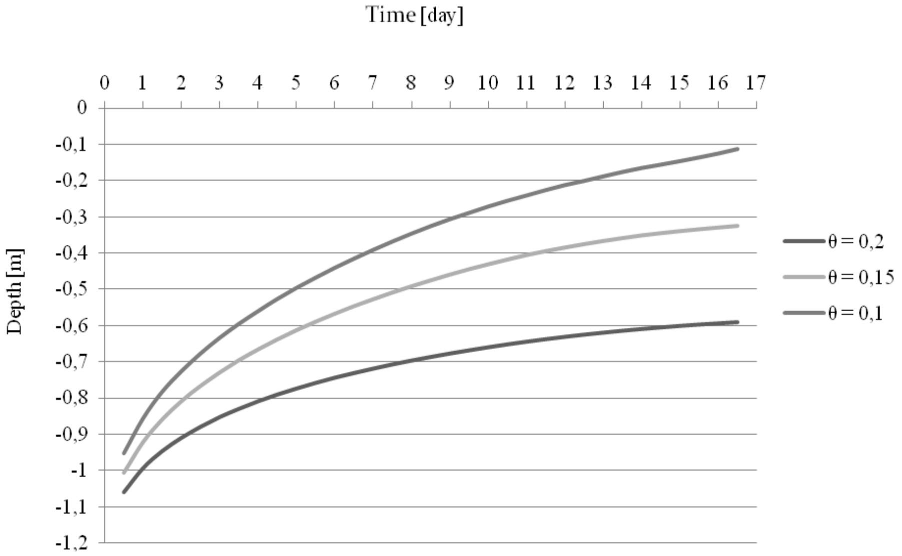
The results of numerical calculations presented indicate that the variations of moisture in time, and also the variations of water stream in soil occur in different way than in figures 2 and 3. A phenomenon of moisture penetration upwards is modelled. Moisture (θ) changes occur more slowly, and the gradient $\frac{\partial\theta}{\partial z}$ is smaller. The water is additionally drawn by plant roots. This activity becomes apparent when the value of moisture near the roots exceeds θ = 0.08. That mechanism was expressed in the mathematical model by the equation (5). After about 16 days, the flows and moisture variations in soil become relatively stable.

The results of numerical calculations presented indicate that the variations of moisture in time, and also the variations of water stream in soil occur in different way than in figures 2 and 3. A phenomenon of moisture penetration upwards is modelled. Moisture (θ) changes occur more slowly, and the gradient $\frac{\partial\theta}{\partial z}$ is smaller. The water is additionally drawn by plant roots. This activity becomes apparent when the value of moisture near the roots exceeds θ = 0.08. That mechanism was expressed in the mathematical model by the equation (5). After about 16 days, the flows and moisture variations in soil become relatively stable.



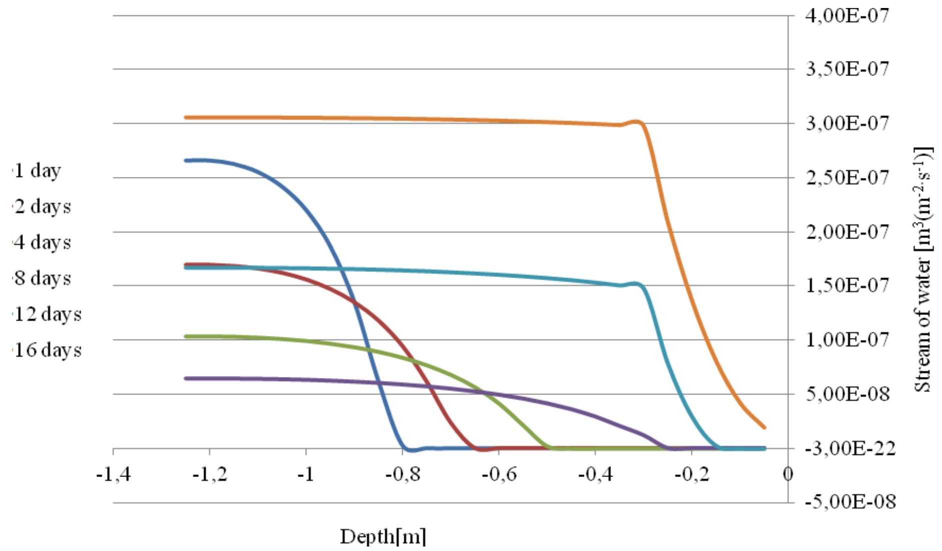
Source: Own study / Źródło: badania własne

Fig. 2. A stream of water in the analyzed soil layer
Rys. 2. Strumień wody w analizowanej warstwie gleby



Source: Own study / Źródło: badania własne

Fig. 3. Changes in soil moisture in time ($K_s = 1.157 \cdot 10^{-6}$, $\gamma = 0.2$ and $n_g = 7.0$)
Rys. 3. Zmiany wilgotności gleby w czasie ($K_s = 1{,}157 \cdot 10^{-6}$, $\gamma = 0{,}2$, oraz $n_g = 7{,}0$)



Source: Own study / Źródło: badania własne

Fig. 4. A stream of water in the analyzed soil layer
Rys. 4. Strumień wody w analizowanej warstwie gleby

5. Summary and conclusions

The mathematical model of water flow in soil presented in this paper, along with the computer program developed, allows modelling of such properties as soil saturating with moisture, capillary rise, precipitation or water uptake by plant roots. By proper selection of empirical ratios for individual soil types it is possible to analyse the soil irrigation level and, if possible, taking certain measures (e.g. irrigation or improvement of drainage system). By adopting proper boundary conditions, the model analysed allows evaluation of the effectiveness of a drainage system. The mathematical model and computer program will be extended by additional equations and procedures allowing modelling of nitrogen migration in soil.

The studies performed allow formulation of the following conclusions:

1. The results obtained and their physical sense, as well as balancing of water streams, indicate the possibility of applying the model to analyse complex hydraulic phenomena in soil.
2. By individual adopting of ratios and the range of their variability for a specific soil and profile of soil, the mathematical model and computer program developed are of universal character. The authors have built a laboratory set and are conducting research into physical properties of selected soil profiles.
3. The model will be further developed to be used for studying of nitrogen migration in soil.

6. References

[1] Chalfen M.: matematyczny model nieustalonego ruchu wód podziemnych z uwzględnieniem obiektów melioracyjnych oraz ujęć wody, Zeszyty Naukowe Akademii Rolniczej we Wrocławiu, 1990 Nr 192, s. 25-38.
[2] Feddes R.A., Bresler E., Neuman S.P.: Field test of a modi-

fied numerical model for water uptake by root systems, water Resources research, Vol.10, No.6, 1974, s. 1199-1206.
[3] Fletcher C.: Computational techniques for fluid dynamics. 1. Fundamental and general techniques. Springer Verlag, Berlin, 1991.
[4] Kowalik P.: Ochrona środowiska glebowego, PWN Warszawa, 2012.
[5] Pospolita J.: Numerical analysis of viscoelastic fluid flux trough the orifice, Archiwum Mechaniki, Warszawa 1991, 43,6, pp.743-751.
[6] Potter C.: Metody obliczeniowe fizyki. Fizyka komputerowa, PWN, Warszawa 1982.
[7] Samarski A.A., Nikołajew J.S.: Metody rozwiązywania równań siatkowych, PWN, Warszawa 1988.
[8] Szulczewski W.: Modelowanie migracji zanieczyszczeń w nienasyconych gruntach i glebach, Zeszyty Naukowe Akademii Rolniczej we Wrocławiu Nr 466, Wrocław 2003.
[9] Szulczewski W.: Modelowanie zmian uwilgotnienia gleby w strefie niepełnego nasycenia, Zeszyty Naukowe Akademii Rolniczej we Wrocławiu, 1990, Nr 192, s. 87-97.
[10] Szymkiewicz A.: Modelling water flow in unsaturated porous media – accounting for nonlinear permeability and material heterogeneity, Springer, 2013.
[11] Uggla H. Gleboznawstwo rolnicze, PWN, Warszawa 1981.
[12] Wosten J.H.M., Van Genuchten M.Th.: Using texture and other soil properties to predict the unsaturated soil hudraulic function, Soil Science American Journal, 1988, Vol. 52, pp. 1762-1770.