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OPTIMIZATION OF REDUCTOR GEAR IN THE HYDROMECHANICAL POWER TRANSMISSION SYSTEM OF A FARM TRACTOR

Summary

In the hydromechanical power transmission system of agricultural tractors is mostly used several speed mechanical gear transmission to change the speed range, so-called reducer. The reason for its introduction is to increase the energy efficiency of the power transmission system. This efficiency depends on the structurally proposed gear ratios in reduction gear. The paper presents the method for determining the optimal gear ratios of this reducer. This method was applied to the numerical example for two-speed reducer.
***Key words**: agricultural tractor, hydromechanical power transmission system, optimal gear ratio*

DOBÓR OPTYMALNEGO PRZEŁOŻENIA REDUKTORA W HYDROMECHANICZNYM UKŁADZIE NAPĘDOWYM CIĄGNIKA ROLNICZEGO

Streszczenie

W hydromechanicznym układzie napędowym ciągników rolniczych znajduje się przeważnie kilkubiegowa przekładnia mechaniczna służąca do zmiany zakresu prędkości jazdy, tak zwany reduktor. Jego funkcją jest zwiększenie sprawności energetycznej układu napędowego. Sprawność ta zależy od przyjętego konstrukcyjnie stosunku między wartościami przełożeń biegów reduktora. W pracy przedstawiono metodę doboru optymalnych przełożeń biegów reduktora. Zastosowanie metody zilustrowano przykładem optymalizacji przełożenia reduktora dwubiegowego.
***Słowa kluczowe**: ciągnik rolniczy, przekładnia hydromechaniczna, optymalne przełożenia reduktora*

1. Introduction

Parallel hydromechanical power transmission system (Fig. 1) has the characteristics of the two transmissions: hydrostatic transmission and mechanical gear transmission. The hydrostatic transmission provides an important advantage: continuously variable ratio of hydromechanical transmission system. However, the efficiency of hydrostatic transmission for part of input power, is relatively low. Mechanical gear transmission for rest of input power has a high efficiency and improves the overall efficiency of the hydromechanical transmission [4]. It is therefore desirable that the part of energy transferred through mechanical way should be the greatest. The part of energy transferred trough mechanical way depends on the current ratio of the hydromechanical transmission system (continuously adjustable), and increases with the hydromechanical transmission ratio (or overall ratio drive system or vehicle speed) from zero at start driving, up to 100% for highest transmission ratio (in this paper the transmission ratio is defined as the quotient of the angular output velocity to the angular input velocity). Hydromechanical power transmission system of a farm tractor provides continuously variable tractor speed, from 0 to the highest speed (from 40 to 60 km·h^{-1} - depending on the tractor manufacturer). It means that a lot of tractor works in agriculture, performed at a speed driving eg. below 12 km·h^{-1}, causes low hydromechanical transmission ratio. It is related to the low part of energy transferred trough mechanical way. To increase the efficiency of the hydromechanical power transmission system behind the hydromechanical transmission, the additional reducer is installed. For instance, there is two-speed reducer in Fendt Vario tractor. Higher gear ratio is used to work at high speed driving. It is characterized by higher value ratios, enabling the achievement of absolutely the highest speed. On a lower gear with the less value ratio, highest speed is about twice less, but during this speed all energy is transferred trough mechanical way.

Farm tractor is used to various works during the all year. Various works cause different tractor speeds, therefore different hydromechanical transmission ratios, which involves specific for each work different part of energy transferred through mechanical way. It is clear that the annual value of this part depends on construction selection of gear ratios reducer. The aim of this study is to clarify the impact of gear ratios reducer on the annual share of energy directed to the mechanical transmission way and to present the methodological basis for maximizing this share.

2. The distribution of power at the input of the hydromechanical transmission system

The part u of power P_m transferred trough mechanical way in hydromechanical power transmission system (Fig. 1), in all input power P_c , depends on the ratio i [2]:

$$u(i) = \frac{1}{i_n} \cdot i, \qquad i \in (i_{n-1}, i_n], \quad i_0 = 0, \tag{1}$$

where:
i – overall gear ratio (between the rotation of the tractor wheels and rotation of the engine),
i_n – overall gear ratio i in n gear reducer position (n = 1,2, ..., s) and fixed the planetary rim gear.

$$i_n = i_p \cdot i_\Sigma \cdot r_n \cdot i_g \cdot i_z, \tag{2}$$

i_p – gear ratio between the central gear and cage of the planetary gear, when fixed the planetary rim gear,
i_Σ – gear ratio between the summing shaft and central gear of the planetary transmission,
r_n – gear ratio in n gear reducer position and the summing shaft,
i_g – main gear ratio,
i_z – wheel reduction gear ratio.

Nowadays there are two- or four- speed reducer gear in the hydromechanical power transmission system of agricultural tractors. However for the sake of general reasoning assumed, that the number of reducer gears s is unrestricted.

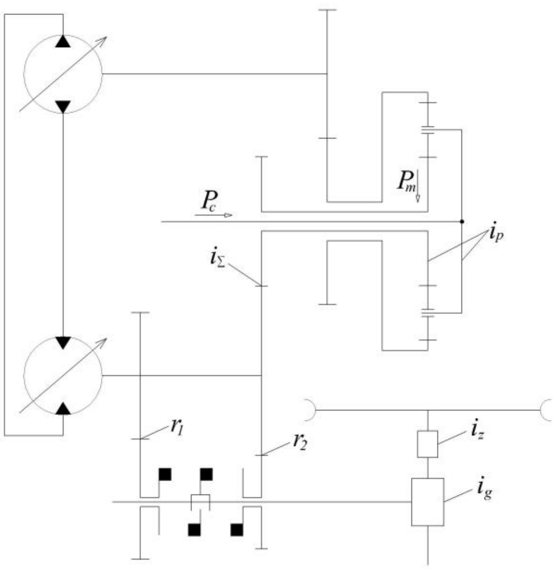

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Fig. 1. Scheme of the hydromechanical power transmission system of agricultural tractors
Rys. 1. Schemat układu napędowego z przekładnią hydromechaniczną

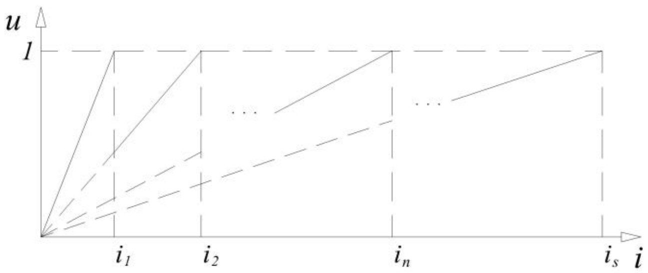

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Fig. 2. The function of part u of energy transferred through mechanical way, depending on the overall gear ratio i
Rys. 2. Zależność udziału u energii przenoszonej gałęzią mechaniczną od przełożenia układu napędowego i

In theory overall gear ratio i may be continuously variable from zero (fixed central gear of the planetary transmission – all power was transferred through hydrostatic way) to the highest value i_n (fixed the planetary rim gear - all power was transferred through mechanical way) in n gear reducer position. Practically, continuously variable transmission ratio carried out between i_{n-1} and i_n, which increases the energy transferred trough mechanical way. Fig. 2 shows the participation u of energy transferred trough mechanical way, depending on the variable overall gear ratio i, with s-speed additional gear reduction unit.

3. The method of optimizing the ratios additional gear reduction unit

Part u of input power, transferred through mechanical way, can be also represented by part of consumed fuel, G_k, attributable to mechanical way of transmission, to the total consumed fuel, G_e, in a given time.

$$u = \frac{P_m}{P_c} = \frac{G_k}{G_e}\,. \tag{3}$$

These interpretation of part u is useful, because it is convenient for analytical and empirical research of intensity needs for different overall gear ratio value during various agricultural works in the year.

Given the variability of the gear ratio, desired for the execution of specific work in the field on an annual basis, should be the gear ratio treated as a random variable I, which takes the values i. Consequently, the part of energy transferred through mechanical way is a random variable U, as a function of the random variable I and i_n parameters:

$$U = \frac{1}{i_n}\cdot I\ ,\ n=1, 2, \ldots, s\ , \tag{4}$$

where: s is the number of reducer speeds.

The aim of gear reducer optimization is to maximize the annual energy transferred through mechanical way of hydromechanical power transmission system. It will be achieved by maximizing the expected value of a random variable U of the equation (4). According to the theory of probability, the expected value of the random variable U as a function of the random variable I, is expressed by the relationship:

$$\mathrm{E}U = \int_{-\infty}^{\infty} u(i)\cdot g(i)\cdot \mathrm{d}i\,, \tag{5}$$

where: $g(i)$ is the probability density function of random variable I. The function $g(i)$ is generated by the weight of consumed fuel.

Taking into account the limits of variation of transmission system ratio, and discrete of $u(i)$ function, according to the equation (1), when changing the i_n parameter (gear change), expected value EU should be written as follows:

$$\mathrm{E}U = \sum_{n=1}^{s}\left[\frac{1}{i_n}\cdot\int_{i_{n-1}}^{i_n} i\cdot g(i)\cdot \mathrm{d}i\right],\ \text{where } i_0=0. \tag{6}$$

Expression (6) is the objective function in the optimization of gear ratio, and the i_n parameters are decision variables. It should be noted, that this function receives its maximum with respect to decision variables, in the interval (0, i_s). This function is additive. It is enough to equate to zero the first partial derivatives of this function (with respect to the subsequent decision variables), to determine the optimum solution:

$$\frac{\mathrm{dE}U}{\mathrm{d}i_n} = 0\ ,\ \text{where } n=1, 2, \ldots s\text{-}1. \tag{7}$$

Designated under the condition (7) an optimal solution lies in the following system of s-1 equations with s-1 unknown values $i_n^{\#}$:

$$\int_{i_{n-1}^{\#}}^{i_n^{\#}} i \cdot g(i) \cdot \mathrm{d}i = \left(1 - \frac{i_n^{\#}}{i_{n+1}^{\#}}\right) \cdot i_n^{\#2} \cdot g(i_n^{\#}) \,, \tag{8}$$

n=1, 2, … s-1

wherein the $i_0^{\#}=0$, $i_s^{\#}=i_s$.

The system of equations (8) shows that the optimum values of the decision variables depend only on the probability density function $g(i)$, and the highest value of the transmission system ratio i_s , which means highest gear reducer.

4. Illustration of method

The following shows the method for example single-speed reducer (s=1) and two-speed reducer (s=2). Hypothetically was assumed the parabolic shape of probability density function for gear ratio of tractor drive system, expressed by the equation:

$$g(i) = -\frac{6}{i_s^3} \cdot i^2 + \frac{6}{i_s^2} \cdot i \,, \quad i \in (0, i_s]. \tag{9}$$

Single-speed reducer (s=1)

The introduction of the expression (9) into relationship (6) and perform calculations gives the result:

$$\mathrm{E}U = \frac{1}{2}. \tag{10}$$

This average value of the share of energy transferred through mechanical way could be predicted, because the transformation of the random variable I in the random variable U is linear, and the function g (i) - symmetrical.

Two-speed reducer (s=2)

Taking into account in the overall solution (8) form (9) of the function $g(i)$ we receive the result:

$$i_1^{\#} = \frac{21 - \sqrt{57}}{24} \cdot i_2 \,. \tag{11}$$

Introduction quotient

$$w^{\#} = \frac{i_1^{\#}}{i_2} \tag{12}$$

can result (11) written as follows:

$w^{\#}$=0,560. (13)

From the equation (2) shows that this quotient at the same time determines the optimal proportion between lower and higher reducer ratio:

$$\frac{r_1^{\#}}{r_2} = w^{\#} \,. \tag{14}$$

Expected value EU, calculated according to the formula (6) for the probability density function (9), is

$$\mathrm{E}U = \frac{1}{2} + \left(3 \cdot w^2 - 7 \cdot w + 4\right) \cdot \frac{w^2}{2} \,,$$

where $w = \frac{i_1}{i_2}$. (15)

Introduction to the equation (15) the optimal solution (13) gives the result:

$(\mathrm{E}U)_{\max}$=0,660. (16)

A graphic illustration of equation (1), (9) and (15) for the two-speed reducer shown sequentially in Fig. 3.

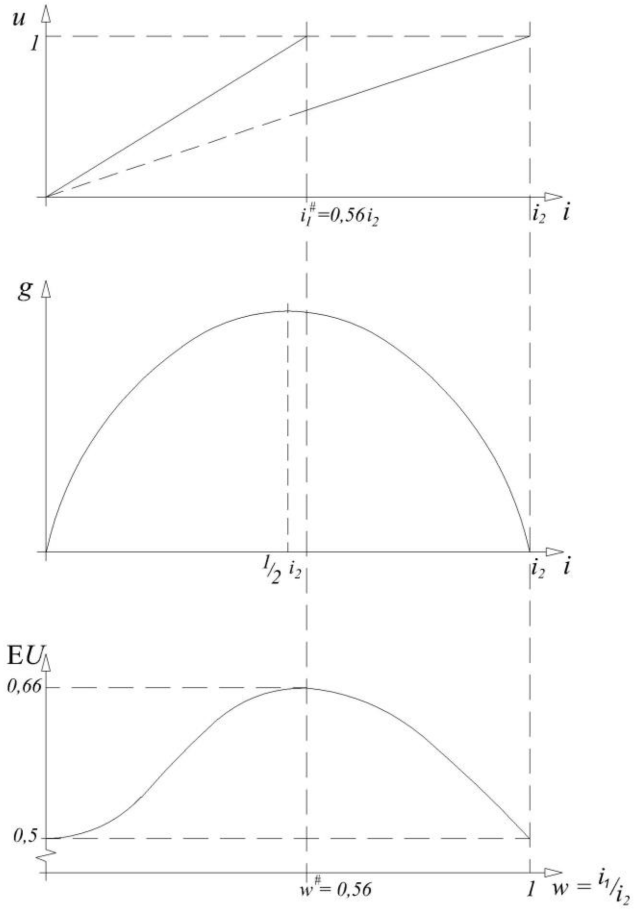

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Fig. 3. The graphic form of equation (1), (9) and (15) for the conditions of example (s = 2)
Rys. 3. Forma graficzna równania (1), (9) i (15) dla warunków przykładu (s=2)

In the present example if single-speed reducer was replaced by the two-speed reducer with an optimum ratio it increases the amount of energy transferred through mechanical way by 32%. For exemplary probability distribution of $g(i)$ optimal proportion between the reducer ratios is $w^{\#}$=0,560. In case of increasing the intensity of annual demand for small value of overall gear ratio, corresponding to a typical field work carried out at speeds from 4 to 12 km·h^{-1}, the optimal proportion between the reducer ratios decreases. For example for Fendt Vario tractors the value of proportions is generally slightly less than ½.

5. Summary

Fundamental to optimizing the gear reducer ratios in hydromechanical power transmission system of a farm tractor is the form of the function $g(i)$. This function can be replaced by a discrete theoretical distribution or empirical distribution. However, the objective function will be discrete, requiring the use of appropriate numerical procedures in order to determine the optimal solution. Obtaining empirical probability distribution of gear ratio required the registration of fuel consumption in each compartments class of gear ratio. The expression of the elementary probability of gear ratio by relative weight of consumed fuel, according to the axiomatic definition of probability, is similar to the method of characterizing engine operating states using so-called time density.

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